Ex.24 解: 三阶矩阵A 有三个不同的特征值, 因此, 对应的特征向量$\alpha_1,\alpha_2,\alpha_3$ 线性无关, 记

$$P=(\alpha_1,\alpha_2,\alpha_3),\ \Lambda=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}.$$

则有$A=P\Lambda P^{-1}$, $A^{100}=P\Lambda^{100}P^{-1}$. 又

$$P^{-1}=\begin{pmatrix}-5&4&-6\\-3&2&-3\\-1&1&-1\end{pmatrix}.$$

所以,

$$\begin{aligned}A=P\Lambda P^{-1}\ &=\begin{pmatrix}1&-2&0\\0&-1&3\\-1&1&2\end{pmatrix}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}\begin{pmatrix}-5&4&-6\\-3&2&-3\\-1&1&-1\end{pmatrix}\\&=\begin{pmatrix}-11&8&-12\\-3&2&-3\\8&-6&9\end{pmatrix}.\end{aligned}$$

$$\begin{aligned}A=P\Lambda^{100}P^{-1}\ &=\begin{pmatrix}1&-2&0\\0&-1&3\\-1&1&2\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}\begin{pmatrix}-5&4&-6\\-3&2&-3\\-1&1&-1\end{pmatrix}\\&=\begin{pmatrix}1&0&0\\3&-2&3\\2&-2&3\end{pmatrix}.\end{aligned}$$